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SYNCHROTRON-UNDULATOR RADIATION IN A NON-UNIFORM MAGNETIC FIELD

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Abstract. A study is made of a charged particle radiation in a non-uniform magnetic field under the conditions when the particle motion trajectory is a curved helix. We consider the case when a pitch angle is less than the reversed value of the relativistic factor. The spectral and angular distribution of the radiation consists of two components separated in frequency. One component is a curvature radiation, and another shows the properties common with undulator radiation.

1. Introduction

As is well known, a radiation spectrum of relativistic charged particles in a magnetic field depends on the angle α between the direction of the particle velocity vector and a line tangent to the field line [1]. If the angle is much greater than the reversed relativistic factor $\gamma^{-1} = \sqrt{1-\beta^2}$, where $\beta = \upsilon/c$ is the particle velocity, then the radiation spectrum has a synchrotron nature, i.e., the peak falls on high harmonics. If the angle is much less than γ^{-1}, then the undulator-type radiation is generated, i.e., the first harmonic is chiefly radiated, and the radiation polarization is almost completely circular. As a rule, in analyzing the relativistic particle radiation in space magnetic fields, the radiation spectrum is assumed to have either a synchrotron or undulator nature (see, e.g., [12]). Meanwhile, the magnetic field lines are commonly curved, consequently, a particle moves along the curvilinear helix, the axis of which, exact to the Alfven drift, coincides with the magnetic field line. Then three different types of radiation can be distinguished: the synchrotron radiation dependent on the helix radius; the undulator radiation when $\gamma^{-1} \geq \alpha$, and the so-called curvature radiation, which is due to the particle motion on the average along the circle arc. Its radius equals the local radius of the field line curvature. An area of small angles exists between extreme values of the angle α . in which both the undulator mechanism of radiation and the curvature radiation play a substantial role [3]. In the present paper we discuss the properties of such mixed radiation.

2. Discussion

Let a relativistic charged particle move in a non-uniform magnetic field. Assume that the size of tile trajectory area from which tile particle radiates In the given direction is much less than the local curvature radius of the magnetic field line. The charge of the curvature radius within the particle trajectory may be neglected. Then the field line in the space area under consideration can be replaced by the circle arc, the radius of which is designated *b* ρ, and the magnetic intensity H can be approximated by function $H = H_0 / \sqrt{x^2 - y^2}$. Here the axis z of Cartesian coordinates is orthogonal to the plane locally in contact with the magnetic field line. The solution of the equation of motion in such a field in a small oscillation approximation can be written as

$$\begin{aligned} x &= [p + r\,sin(\omega_0 t - \phi)]cos\,\omega_p t, \\ y &= [p + r\,sin(\omega_0 t - \phi)]sin\,\omega_p t, \\ z &= \upsilon_{dr} t - r\,cos(\omega_0 t - \phi), \end{aligned} \tag{1}$$

where ϕ is the arbitrary initial phase. We take the coordinate origin to be In the local center of the field line curvature. The motion defined by equations (1) is a superposition of three motions: namely the rotation along the circle, with a radius $r \ll \rho$ in a plane containing the z-axis; the motion along the circle, with a large radius

ρ, around the z-axis, and a slow Alfven drift towards tile z-axis, which is related to the magnetic field non-uniformity. The velocity of this drift is derived from the generally known formula [l]:

$\upsilon_{dr} = \frac{1}{\omega_0 p}\left[\upsilon_\perp^2 + \frac{1}{2}\upsilon_{||}^2\right]$, where $\upsilon_\perp$ and $\upsilon_{||}$ *we* defined by the initial conditions and are, correspondingly, the particle velocity components orthogonal and tangent to tile field line. Frequencies ω_0, ω_ρ and the radius of helix r are determined by equalities:

$$\omega_0 = \frac{eH}{mc\gamma}, \quad \omega_\rho = \frac{\upsilon_{||}}{\rho}, \quad r = \frac{\upsilon_\perp mc\gamma}{eH}. \tag{2}$$

Here c and m are the charge and mass of a particle, respectively, H is the magnetic intensity module, which is considered to be constant in the neighborhood of the trajectory, c is the speed of light. The angle between the velocity vector and the line tangent to the magnetic field line is taken to be much less than γ^{-1}, i.e.:

$$k = \frac{\upsilon_\perp}{\upsilon_{||}}\gamma < 1. \tag{3}$$

In the theory of the undulator radiation the parameter k determines the shape of radiation spectrum. When tile condition (3) is satisfied, only the first harmonic is generated. With increasing k the radiation increases at multiple frequencies (see, e.g. [4]).

Close by zero time t=0 the radiation is generated as a narrow cone in the neighborhood of the positive direction of the z-axis. Generally speaking, the spectral-angular distribution of radiation is not axial - symmetrical with respect to the z-axis. When turning round the z-axis, it is repeated through the angle $\Delta=\omega_\rho/\omega_0$. This angle is assumed to be much less than an opening of the radiation cone γ^{-1}. Let $N=\omega_0/\gamma\omega_\rho$. Then N»1 . The value N coincides in an order of magnitude with tile number of revolutions, which the particle manages to make over the radiation time in the given direction n. If N »1, the radiation can be regarded as axial-symmetrical. In this case, without *low* of generality, one can calculate a unit vector directed to all observer arid lying in the yz-plane.

$$\vec{n} = (0, cos\,\chi, sin\,\chi), \chi \langle\langle 1.$$

Single unit vectors of polarization components are found by the following way:

$$e_\sigma = (-1,0,0), e_\pi = \left[e_\sigma \bar{n}\right] = (0, sin\,\chi, -cos\,\chi)$$

Spectral expansion of the radiation field can be presented as [1]

$$E_j(\omega) = \frac{ei\omega}{cR}e^{ikR}\int_{-\infty}^{\infty}\beta_j(t)e^{i(\omega t - \vec{k}\vec{r})}dt, \quad j = \sigma, \pi, \tag{4}$$

where R is the distance between the charge and tile observer, $\vec{k} = \vec{n}\omega / c$ is the undulator vector, and $\vec{r}$ is the particle radius-vector. The radiation in the direction $\vec{n}$ is generated during the time of an order of $\omega_\rho t \sim \gamma^{-1}$.

This allows us to expand the trigonometric functions with argument $\omega_\rho t$ into a row. Keeping terms to the 3rd order of smallness for $\omega_\rho t$, χ and r/ρ, we find

$$\left(\omega t - \vec{k}\vec{r}\right) \approx \omega t\left[\frac{1}{2}\left(\gamma^{-2} + \chi^2\right) - \frac{r}{p}\sin(\omega_0 t - \phi) - \beta_{dr}\chi\right] + \frac{r}{p}\omega\chi\cos(\omega_0 t - \phi) \tag{5}$$

Let us estimate the order of magnitude of terms ill the last expression. As is shown later, the nature of radiation depends on the frequency and ω_ρ relation. The characteristic frequency of radiation generated due to the synchrotron mechanism lies in the neighborhood of $\omega_{sr} \sim \omega_\rho\gamma^3$, and the characteristic frequency of the undulator radiation makes up the value of an order of $\omega_{ur} \sim \omega_0\gamma^2$. The undulator frequency is much greater than the synchrotron frequency, as predicted, for $\omega_{sr}/\omega_{sr} \sim N$. Terms with amplitude of r$\omega t/\rho$ and $r\omega\chi/c$: in expression (5) have the same order of magnitude, which is equal to ~ $\upsilon_\perp\,\gamma/c$ ~ k «1 at an undulator frequency. Thus, these terms can be neglected at frequencies characteristic of the undulator radiation, and all the more at frequencies of the synchrotron radiation. We shall introduce a designation $\chi_d=\chi-\beta_{dr}$ and consider that in an ultra-relativistic approximation $\omega_p\rho\approx c$. Then the relation (5) will be

$$\left(\omega t-\vec{k}\vec{r}\right)\approx\frac{\omega t}{2}\left(\gamma^{-2}+\chi_d^2-\beta_{dr}^2\right)+\frac{1}{6}\omega\omega_p^2t^3. \tag{6}$$

In a similar way, expanding $\beta_j(t)$ in (4) we find $E_\sigma(\omega)$

$$\beta_\sigma(t)=\omega_pt-\frac{r\omega_0}{c}cos(\omega_0t-\phi),\quad \beta_\pi(t)=\chi d-\frac{r\omega_0}{c}sin$$

After an operation of integrating in relation (4) we find, e.g. for

$$E_\sigma(\omega)=\frac{2ei\omega e^{ikR}}{\sqrt{3}cR\omega_p\gamma^2}\left\{i\left(1+\psi^2\right)K_{\frac{2}{3}}(z)-\frac{\omega_0r\gamma}{2c}\left[e^{-i\phi}\sqrt{1+\psi^2+\nu^{-1}}K\right.\right. \tag{7}$$

where

$$\psi=\chi d\gamma,\quad \nu=\frac{\omega}{2\gamma^2\omega_0},\quad z=\frac{\omega(1+\psi)^{\frac{3}{2}}}{3\gamma^3\omega_p},\quad z_+=\frac{2}{3}\nu N[1 \tag{8}$$

$$F(z_-)=\begin{cases}\dfrac{\pi}{\sqrt{3}}\sqrt{\eta}\left[J_{-\frac{1}{3}}(z_-)+J_{\frac{1}{3}}(z_-)\right], & z_-=\dfrac{2}{3}\nu N[-\eta]^{\frac{3}{2}},\\ \sqrt{\eta}\,K_{\frac{1}{3}}(z_-), & z_-=\dfrac{2}{3}\nu N[-\eta]^{\frac{3}{2}},\end{cases} \tag{9}$$

$$\eta=1+\psi^2-\nu^{-1}.$$

It is easy to see that the function $F(z\_)$ taken relatively greater values within an area $\omega\approx 2\omega_0/\phi$. Under these conditions $F(z\_)\sim 1$. In this portion of spectrum $z_+ \gg 1$, consequently, the function $K_{1/3}(z_+) \ll 1$, and it can be ignored. Thus, the polarization components of the radiation field can be written as

$$E_\sigma=\frac{4ie\omega_0e^{ikR}}{\sqrt{3}cR\omega_p}\nu\left[i\left(1+\psi^2\right)K_{\frac{2}{3}}(z)-\frac{1}{2}ke^{i\phi}F(z_-\right. \tag{10}$$

$$E_\pi=\frac{4ie\omega_0e^{ikR}}{\sqrt{3}cR\omega_p}\nu\left[\psi\sqrt{1+\psi^2}K_{\frac{1}{3}}(z)-\frac{1}{2}ike^{i\phi}F(z_-\right. \tag{11}$$

The spectral-angular distribution of energy E_j radiated by a particle is calculated from the formula [1]

$$\frac{d\varepsilon_j}{d\Omega d\omega}=\frac{cR^2}{4\pi^2}\left|E_j(\omega)\right|^2. \tag{12}$$

Substituting expressions (10) and (11), we find

$$\begin{aligned}\frac{d\varepsilon_\sigma}{d\Omega\, d\omega}&=\frac{4e^2\omega_0^2\nu^2}{3c\pi^2\omega_p^2}\left[\left(1+\psi^2\right)^2K_{\frac{2}{3}}^2(z)+k\,sin\,\phi\,K_{\frac{2}{3}}(z)F(z_-)+\right.\\ \frac{d\varepsilon_\pi}{d\Omega\, d\omega}&=\frac{4e^2\omega_0^2\nu^2}{3c\pi^2\omega_p^2}\left[\left(1+\psi^2\right)^2K_{\frac{1}{3}}^2(z)+k\,sin\,\phi\,K_{\frac{1}{3}}(z)F(z_-)+\right.\end{aligned} \tag{13}$$

The first term in formulae (13) coincides with expressions for a synchrotron radiation of a particle moving along the circle with radius ρ and describes the curvature radiation. The third containing the function $F^2(z\_)$, is proportional to the helix radius squared and, consequently is the undulator-type radiation. The second term, proportional to $\sin\phi$, describes, in a particular sense, the "interference" of the fields of the synchrotron and undulator types.

Functions $\nu K_{2/3}(z)$ and $\nu K_{2/3}(z\_)$ show a peak in the neighborhood of $z\sim 1$, i.e., at frequency $\omega_{sr}\sim\Omega\gamma^3$. The function $\nu F(z\_)$ differs essentially from zero in the area $z\_\sim 1$. The corresponding reduced frequency ν_{ur} can be calculated from

$$N\nu_{ur}\left|1+\psi^2-\frac{1}{\nu_{ur}}\right|^{\frac{3}{2}}\sim 1.$$

Hence $\nu_{ur}\sim\dfrac{1}{1+\psi^2}\pm\dfrac{1}{N}$

The reduced angle ψ has an order of unity, consequently, $\nu_{ur}\sim 1$.Thus the undulator-type radiation falls on a respectively narrow band of frequencies with relative width $\Delta\omega/\omega\sim N^{-1}$ in the neighborhood of frequency $\sim 2\omega_0\gamma^2$. As $\omega_{ur}/\omega_{sr}\sim N \gg 1$, the synchrotron and undulator radiation turned to be far separated in frequency. The spectrum in the intermediate area of frequencies $\omega_{sr}\ll\omega\ll\omega_{ur}$ is described by the second term in formulae (13). This term is proportional to $\sin\phi$ and depends greatly on tile initial phase of motion. In actual conditions we must deal with an ensemble of particles and average the radiation properties by initial parameters. As a result of such averaging the second term in formulae (13) vanishes.

Figures 1 and 2 show the spectral distribution of radiated energy as a function of reduced frequency ν for polarization component σ an angle $\chi_d = 0$ for different values of the initial phase ϕ and parameters N=20, K=0,1. As shown in Fig. 1, the radiation spectrum is distinctly subdivided into the synchrotron and undulator portions. The undulator portion is related to frequency $\nu=1$. Fig. 2 demonstrates the influence of the initial phase upon tile shape of radiation spectrum: the character of oscillations in the intermediate area of frequencies is essentially dependent upon the value ϕ. The impact of the pitch-angle α on the shape of spectrum is seen from formulae (13). With decreasing α, the undulator parameter k decreases and hence the contribution of the undulator radiation also decreases.

Formulae for the Fourier components of the radiation field (10) and (11) show that the phases of σ- and π-components of the undulator portion of radiation differ by tile value of $\pi/2$, with amplitude being the same. Consequently, this portion of radiation has a circular polarization. It must be emphasized that the elliptical polarization of the curvature radiation described by the first term of formulae (10) and (11) reverses its sign when ψ is substituted for $-\psi$, whereas the undulator portion of radiation has the polarization of the helicity at all angle ranges within the radiation cone. This property call be used to determine the character of particle motion on the basis of its radiation.

Now we shall find the radiation circular polarization components. They are derived from the formula (12), where $E_+(\omega)$ and $E_-(\omega)$, i.e., amplitudes of the right hand and left-hand circular polarization, respectively, should be inserted instead of $E_j(\omega)$. Circular polarization components call be expressed through the linear polarization components [5]:

$$E_+=\frac{E_\pi-iE_\sigma}{\sqrt{2}},\quad E_-=\frac{E_\pi-iE_\sigma}{\sqrt{2}}.$$

As a result, we have the following expressions for the spectral-angular distribution

$$\frac{d\varepsilon_+}{d\Omega d\omega}=\frac{2e^2\omega_0^2\nu^2}{3c\pi^2\omega_\rho^2}[\,\psi\sqrt{1+\psi^2}K_{\frac{1}{3}}(z)+\left(1+\psi^2\right).$$
$$\frac{d\varepsilon_-}{d\Omega d\omega}=\frac{2e^2\omega_0^2\nu^2}{3c\pi^2\omega_\rho^2}[\,f^2(\omega)+2k\sin\phi f(\omega)F(z_-)+k^2 \qquad (14)$$

Here $f(\omega) = \psi\sqrt{1+\psi^2}K\frac{1}{3}(z) - (1+\psi^2)K\frac{2}{3}(z).$

Thus, the synchrotron portion of radiation has both the right-hand and left-hand polarization components, i.e., it is elliptically polarized, whereas the undulatur radiation is presented only In the left-hand polarized component.

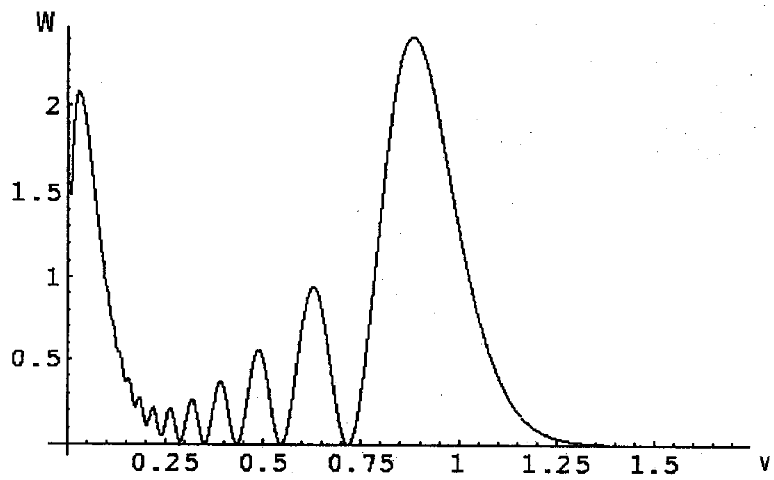

Fig. 1. Radiation spectrum of component σ at an angle $\chi_d = 0$. The initial phase $\phi = 0$.

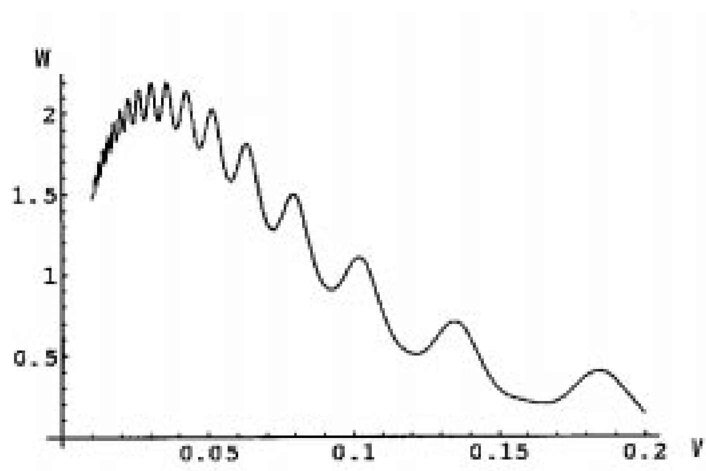

Fig. 2. Synchrotron portion of radiation spectrum of component σ at an angle $\chi_d = 0$. The initial phase $\phi = \pi/6$.

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