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ASYMPTOTIC RESULTS FOR EMDEN-FOWLER SYSTEM OF THE DIFFERENTIAL EQUATIONS

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ABSTRACT. We shall establish sufficient conditions for an asymptotic representation of solutions of the system of differential equations of the Emden-Fowler type

$$u_i' = a_i(t)|u_{3-i}|^{\lambda_i}\, sgn\, u_{3-i} \quad (i = 1, 2).$$

The obtained results generalize results of Kiguradze I.T. and Chanturiya T.A. for the second order differential equation of the Emden-Fowler type.

Introduction

The differential equation

$$u''(t) = a(t)|u(t)|^{\lambda}\, sgn\, u(t), \tag{1}$$

where $\lambda \neq 1$, is known in tile literature as the equation of the Emden-Fowler type. The equation of this type for the first time has attracted attention around the turn of the century with earlier theories concerning gaseous dynamics in astrophysics. For a summary of some important historical developments concerning this equation we refer to the papers by Kiguradze I.T. [3] and Wong J.S.W. [8].

The oscillatory and nonoscillatory behavior and asymptotic properties of solutions of the Emden-Fowler equation have been considerably investigated by many authors. For a survey on such results we refer to the book of Kiguradze I.T. and Chanturiya T.A. [6], where an extensive bibliography is contained. For tile system of differential equations of the Emden- Fowler type

$$u_1' = a_1(t)|u_2|^{\lambda_1}\, sgn\, u_2, \qquad u_2' = -a_2(t)|u_1|^{\lambda_2}\, sgn\, u_1, \tag{2}$$

fairly less research work have been .done. The asymptotic properties of solutions of the system *('2)* have been considered in several papers ([1], [5], [6]).

Denote the set of functions which are absolutely continuous on each finite segment of the interval [0, +∞) by $\widetilde{C}_{loc}$([0, +∞)). It will be assumed that the functions $a_i \in \widetilde{C}_{loc}$([0, +∞) *(i* = 1,2) are nonnegative and $\lambda_i > 0$, *(i* = 1,2).

1. Previous results.

Mirzov D.D. has proved in [5] the following theorems.

Theorem 1.1. *Suppose, that the following conditions hold*

$$\lim_{t\to+\infty} \beta(t) = 0, \tag{1.1}$$

$$\int_0^{+\infty} |d\beta(t)| < +\infty, \tag{1.2}$$

where

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$$\beta(t) = -\frac{1}{2+\lambda_1+\lambda_2}\left(\frac{a_1(t)}{a_2(t)}\right)^{-1-\frac{1+\lambda_2}{2+\lambda_1+\lambda_2}}\frac{1}{a_2(t)}\left(\frac{a_1(t)}{a_2(t)}\right)'. \tag{1.3}$$

Then, any nontrivial solution ($u_1(t)$, $u_2(t)$) of the system (2) satisfies the condition

$$\lim_{t\to+\infty}\rho(t) = \rho_0 \in (0,+\infty), \tag{1.4}$$

or for all $t \geq 0$

$$\rho(t) \leq 2^{\frac{(1+\lambda_1)(1+\lambda_2)}{\lambda_1\lambda_2-1}}(1+\lambda_1)^{\frac{1+\lambda_2}{\lambda_1\lambda_2-1}}(1+\lambda_2)^{\frac{1+\lambda_1}{\lambda_1\lambda_2-1}}\left(\int_t^{+\infty}|d\beta(\tau)|\right)^{\frac{(1+\lambda_1)(1+\lambda_2)}{\lambda_1\lambda_2-1}} \tag{1.5}$$

for $\lambda_1\lambda_2 > 1$,

$$\rho(t) \geq 2^{\frac{2+\lambda_1+\lambda_2+(1+\lambda_1)(1+\lambda_2)}{\lambda_1\lambda_2-1}}(1+\lambda_1)^{\frac{1+\lambda_2}{\lambda_1\lambda_2-1}}(1+\lambda_2)^{\frac{1+\lambda_1}{\lambda_1\lambda_2-1}}\left(\int_t^{+\infty}|d\beta(\tau)|\right)^{\frac{(1+\lambda_1)(1+\lambda_2)}{\lambda_1\lambda_2-1}} \tag{1.6}$$

for $\lambda_1\lambda_2 < 1$, *where*

$$\rho(t) = \left(\frac{a_2(t)}{a_1(t)}\right)^{\frac{1+\lambda_2}{2+\lambda_1+\lambda_2}}\frac{|u_1(t)|^{1+\lambda_2}}{1+\lambda_2} + \left(\frac{a_1(t)}{a_2(t)}\right)^{\frac{1+\lambda_1}{2+\lambda_1+\lambda_2}}\frac{|u_2(t)|^{1+\lambda_1}}{1+\lambda_1} \tag{1.7}$$

Theorem 1.2. *Let the conditions of Theorem* 1.1. *be fulfilled. Then, any nontrivial solution* ($u_1(t)$, $u_2(t)$) *of the, system* (2), *which satisfies the condition* (1.4), *may be represented in the following form*

$$\begin{aligned} u_1(t) &= \left(\frac{a_1(t)}{a_2(t)}\right)^{\frac{1}{2+\lambda_1+\lambda_2}}(\rho(t))^{\frac{1}{1+\lambda_2}}\omega_1(\alpha(t)), \\ u_2(t) &= \left(\frac{a_2(t)}{a_2(t)}\right)^{\frac{1}{2+\lambda_1+\lambda_2}}(\rho(t))^{\frac{1}{1+\lambda_1}}\omega_2(\alpha(t)), \end{aligned} \tag{1.8}$$

where the function $\rho(t)$ defined by (1.7) *and the function $\alpha(t)$ satisfy the, following condition*

$$\lim_{t\to+\infty}\frac{\alpha(t)}{\int_0^t a_2(\tau)\left(\frac{a_2(\tau)}{a_1(\tau)}\right)^{-\frac{1+\lambda_2}{2+\lambda_1+\lambda_2}}d\tau} = \rho_0^{\frac{\lambda_1\lambda_2-1}{(1+\lambda_1)(1+\lambda_2)}} \tag{1.9}$$

and ($\omega_1(t)$,$\omega_2(t)$) is a solution of the problem

$$\omega_1' = |\omega_2|^{\lambda_1}\,sgn\,\omega_2,\ \omega_2' = -|\omega_1|^{\lambda_2}\,sgn\,\omega_1 \tag{1.10}$$

$$\frac{|\omega_1(0)|^{1+\lambda_2}}{1+\lambda_2} + \frac{|\omega_2(0)|^{1+\lambda_1}}{1+\lambda_1} = 1.$$

Theorem 1.3. *Suppose that the conditions* (1.1) *and* (1.2) *are satisfied. Then, for any* $\rho_0 \in$ (0, $+\infty$) *there exists a solution ($u_1(t)$, $u_2(t)$) of the system* (2) *which satisfies the condition* (1.4).

According to Theorem 1.2. and Theorem 1.3. the system (2) possesses solutions which may be represented as (1.8), but the conclusions (1.1) and (1.2) are not enough for correctness of such asymptotic representation for all solutions. For this reason Mirzov D.D. has established in [5] some conclusions which ensure that all solutions of the system (2) may be represented as (1.8).

2. Main results

The aim here is to establish more conclusions which ensure aforementioned asymptotic representation. This results extend the conditions discovered by Kiguradze I.T. [2] and Chanturiya T.A. [7] for the Emden-Fowler equation.

Theorem 2.1. *Let the function* $\frac{a_1(t)}{a_2(t)}$ *be nondecreasing and*

$$f(t)=\left(\frac{a_1(t)}{a_2(t)}\right)^{-\frac{1}{1+\lambda_1}}\frac{1}{a_1(t)}\left(\frac{a_1(t)}{a_2(t)}\right)', \tag{2.1}$$

$$g(t)=\left(\frac{a_1(t)}{a_2(t)}\right)^{-\frac{\lambda_2}{1+\lambda_2}}\frac{1}{a_1(t)}\left(\frac{a_1(t)}{a_2(t)}\right)'. \tag{2.2}$$

If the condition

$$\int_0^{+\infty}|df(t)|<+\infty,\quad \lim_{t\to+\infty}f(t)=0\quad \text{for}\quad \lambda_1\lambda_2>1,$$

$$\int_0^{+\infty}|dg(t)|<+\infty,\quad \lim_{t\to+\infty}g(t)=0\quad \text{for}\quad \lambda_1\lambda_2<1,$$

is valid often any nontrivial solution of the system (2) *may be, represented as* (1.8), *where the function p(t) defined by* (1.7) *and the function* $\alpha(t)$ *satisfy the condition* (1.9) *and* $(\omega_1(t)\omega_2(t))$ *is a solution of the problem* (1.10).

Proof. We divide the proof into two cases. *Case 1.* $\lambda_1\lambda_2$ > I. According to (1.3) and (2.1) we have

$$\beta(t)=-\frac{1}{2+\lambda_1+\lambda_2}\left(\frac{a_1(t)}{a_2(t)}\right)^{\frac{1-\lambda_1\lambda_2}{(1+\lambda_1)(2+\lambda_1+\lambda_2)}}f(t).$$

Since, $1-\lambda_1\lambda_2<0$ and $\frac{a_1(t)}{a_2(t)}$ is the nondecreasing function we find that

$$0\ge\beta(t)\ge-\frac{1}{2+\lambda_1+\lambda_2}\left(\frac{a_1(0)}{a_2(0)}\right)^{\frac{1-\lambda_1\lambda_2}{(1+\lambda_1)(2+\lambda_1+\lambda_2)}}f(t)\to 0,\quad t\to+\infty$$

Therefore, $\lim_{t\to+\infty}\beta(t)=0$. Putting

$$\Lambda_1=\frac{1-\lambda_1\lambda_2}{(1+\lambda_1)(2+\lambda_1+\lambda_2)},$$

we get

$$d\beta(t)=\frac{\lambda_1\lambda_2-1}{(1+\lambda_1)(2+\lambda_1+\lambda_2)^2}\left(\frac{a_1(t)}{a_2(t)}\right)^{\Lambda_1-1}\left(\frac{a_1(t)}{a_2(t)}\right)^{-\frac{1}{1+\lambda_1}}\frac{1}{a_1(t)}\left[\left(\frac{a_1(t)}{a_2(t)}\right)'\right]^2-$$

$$-\frac{1}{2+\lambda_1+\lambda_2}\left(\frac{a_1(t)}{a_2(t)}\right)^{\Lambda_1}df(t). \tag{2.3}$$

Integrating the previous equality over [0, +∞) and using the fact that $\Lambda_1 < 0$ and $\frac{a_1(t)}{a_2(t)}$ is the nondecreasing function we obtain

$$\frac{\lambda_1\lambda_2-1}{(1+\lambda_1)(2+\lambda_1+\lambda_2)^2}\int_0^{+\infty}\left(\frac{a_1(t)}{a_2(t)}\right)^{\Lambda_1-1}\left(\frac{a_1(t)}{a_2(t)}\right)^{-\frac{1}{1+\lambda_1}}\frac{1}{a_1(t)}\left[\left(\frac{a_1(t)}{a_2(t)}\right)^{\prime}\right]^2 dt \le$$

$$\le|\beta(0)|+\frac{1}{2+\lambda_1+\lambda_2}\int_0^{+\infty}\left(\frac{a_1(t)}{a_2(t)}\right)^{\Lambda_1}|df(t)|$$

$$\le|\beta(0)|+\frac{1}{2+\lambda_1+\lambda_2}\left(\frac{a_1(0)}{a_2(0)}\right)^{\Lambda_1}\int_0^{+\infty}|df(t)|<+\infty.$$

From (2.3) we deduce that $\int_0^{+\infty}|d\beta(t)|<+\infty$. Accordingly, the conditions of Theorem 1.1. are satisfied.

Assume that an arbitrary solution $(u_1(t), u_2(t))$of the system (2) satisfies (1.5). From (2.3), we have for $t \ge 0$

$$\int_t^{+\infty}|d\beta(t)|\le\frac{1}{2+\lambda_1+\lambda_2}\left(\frac{a_1(t)}{a_2(t)}\right)^{\Lambda_1}\int_t^{+\infty}|df(\tau)|+$$

$$+\frac{\lambda_1\lambda_2-1}{(1+\lambda_1)(2+\lambda_1+\lambda_2)^2}\int_t^{+\infty}\left(\frac{a_1(\tau)}{a_2(\tau)}\right)^{\Lambda_1-1}\left(\frac{a_1(\tau)}{a_2(\tau)}\right)^{-\frac{1}{1+\lambda_1}}\frac{1}{a_1(\tau)}\left[\left(\frac{a_1(\tau)}{a_2(\tau)}\right)^{\prime}\right]^2 d\tau. \tag{2.4}$$

By (1.1) and (2.3) we find that

$$\frac{\lambda_1\lambda_2-1}{(1+\lambda_1)(2+\lambda_1+\lambda_2)^2}\int_t^{+\infty}\left(\frac{a_1(\tau)}{a_2(\tau)}\right)^{\Lambda_1-1}\left(\frac{a_1(\tau)}{a_2(\tau)}\right)^{-\frac{1}{1+\lambda_1}}\frac{1}{a_1(\tau)}\left[\left(\frac{a_1(\tau)}{a_2(\tau)}\right)^{\prime}\right]^2 d\tau. \le$$

$$\le -\beta(\mathrm{t})+\frac{1}{2+\lambda_1+\lambda_2}\left(\frac{a_1(t)}{a_2(t)}\right)^{\Lambda_1}\int_t^{+\infty}|df(\tau)|$$

and

$$\frac{1}{2+\lambda_1+\lambda_2}\left(\frac{a_1(t)}{a_2(t)}\right)^{\Lambda_1}\int_t^{+\infty}|df(\tau)|\ge\frac{1}{2+\lambda_1+\lambda_2}\left(\frac{a_1(t)}{a_2(t)}\right)^{\Lambda_1}\int_t^{+\infty}df(\tau)=-\beta(t)$$

Combining previous inequalities with (2.4) we obtain

$$\int_t^{+\infty}|d\beta(\tau)|\le\frac{3}{2+\lambda_1+\lambda_2}\left(\frac{a_1(t)}{a_2(t)}\right)^{\frac{1-\lambda_1\lambda_2}{(1+\lambda_1)(2+\lambda_1+\lambda_2)}}\int_t^{+\infty}|df(\tau)| \text{ for } \mathrm{t}\ge 0 \tag{2.5}$$

Since we suppose that (1.5) is valid, we have

$$\rho(t)\le 2^{\frac{(1+\lambda_1)(1+\lambda_2)}{\lambda_1\lambda_2-1}}(1+\lambda_1)^{\frac{1+\lambda_2}{\lambda_1\lambda_2-1}}(1+\lambda_2)^{\frac{1+\lambda_1}{\lambda_1\lambda_2-1}}\left(\frac{3}{2+\lambda_1+\lambda_2}\right)^{\frac{(1+\lambda_1)(1+\lambda_2)}{\lambda_1\lambda_2-1}}$$

$$\left(\frac{a_1(t)}{a_2(t)}\right)^{-\frac{1+\lambda_2}{2+\lambda_1+\lambda_2}}\left(\int_t^{+\infty}|df(\tau)|\right)^{\frac{(1+\lambda_1)(1+\lambda_2)}{\lambda_1\lambda_2-1}}$$

Consequently,

$$\lim_{t\to+\infty}\rho(t)\left(\frac{a_1(t)}{a_2(t)}\right)^{\frac{1+\lambda_2}{2+\lambda_1+\lambda_2}}=\lim_{t\to+\infty}\left(\frac{|u_1(t)|^{1+\lambda_2}}{1+\lambda_2}+\frac{a_1(t)|u_2(t)|^{1+\lambda_1}}{a_2(t)\ 1+\lambda_1}\right)=0$$

However, this is contradictory to

$$\left(\frac{|u_1(t)|^{1+\lambda_2}}{1+\lambda_2}+\frac{a_1(t)}{a_2(t)}\frac{|u_2(t)|^{1+\lambda_1}}{1+\lambda_1}\right)'=\left(\frac{a_1(t)}{a_2(t)}\right)'\frac{|u_2(t)|^{1+\lambda_1}}{1+\lambda_1}\geq 0$$

According to Theorem 1.1. the solution ($u_1(t)$, $u_2(t)$) satisfies the assumption (1.4) and a direct application of Theorem 1.2. yields the desired conclusion.

Case 2. $\lambda_1\lambda_2<1$. According to (1.3) and (2.2) we find

$$0\geq\beta(t)=-\frac{1}{2+\lambda_1+\lambda_2}\left(\frac{a_1(t)}{a_2(t)}\right)^{\frac{\lambda_1\lambda_2-1}{(1+\lambda_2)(2+\lambda_1+\lambda_2)}}g(t)$$

$$\geq-\frac{1}{2+\lambda_1+\lambda_2}\left(\frac{a_1(0)}{a_2(0)}\right)^{\frac{\lambda_1\lambda_2-1}{(1+\lambda_2)(2+\lambda_1+\lambda_2)}}g(t)\to 0,\quad t\to+\infty$$

Consequently, $\lim\limits_{t\to+\infty}\beta(t)=0$. Putting $\Lambda_2=\dfrac{\lambda_1\lambda_2-1}{(1+\lambda_2)(2+\lambda_1+\lambda_2)}<0,$

we obtain

$$d\beta(t)=\frac{1-\lambda_1\lambda_2}{(1+\lambda_2)(2+\lambda_1+\lambda_2)^2}\left(\frac{a_1(t)}{a_2(t)}\right)^{\Lambda_2-1}\left(\frac{a_1(t)}{a_2(t)}\right)^{-\frac{\lambda_2}{1+\lambda_2}}\frac{1}{a_1(t)}\left[\left(\frac{a_1(t)}{a_2(t)}\right)'\right]^2-$$

$$-\frac{1}{2+\lambda_1+\lambda_2}\left(\frac{a_1(t)}{a_2(t)}\right)^{\Lambda_2}dg(t).$$

As in the previous case we conclude that the condition (1.2) is satisfied and

$$\int_t^{+\infty}|d\beta(\tau)|\leq\frac{3}{2+\lambda_1+\lambda_2}\left(\frac{a_1(t)}{a_2(t)}\right)^{\frac{\lambda_1\lambda_2-1}{(1+\lambda_2)(2+\lambda_1+\lambda_2)}}\int_t^{+\infty}|dg(\tau)|\quad\text{for}\quad t\geq 0. \tag{2.6}$$

According to Theorem 1.2. it is enough to prove that an arbitrary solution ($u_1(t)$, $u_2(t)$) of the system (2) satisfies the condition (1.4). Assume the contrary. Applying Theorem 1.1. we deduce that the system (2) possess a solution which satisfies (1.6). From (2.6) we find that

$$\rho(t)\geq 2^{\frac{(1+\lambda_1)(1+\lambda_2)+2+\lambda_1+\lambda_2}{\lambda_1\lambda_2-1}}(1+\lambda_1)^{\frac{1+\lambda_2}{\lambda_1\lambda_2-1}}(1+\lambda_2)^{\frac{1+\lambda_1}{\lambda_1\lambda_2-1}}\left(\frac{3}{2+\lambda_1+\lambda_2}\right)^{\frac{(1+\lambda_1)(1+\lambda_2)}{\lambda_1\lambda_2-1}}$$

$$\left(\frac{a_1(t)}{a_2(t)}\right)^{\frac{1+\lambda_1}{2+\lambda_1+\lambda_2}}\left(\int_t^{+\infty}|dg(\tau)|\right)^{\frac{(1+\lambda_1)(1+\lambda_2)}{\lambda_1\lambda_2-1}},$$

which implies

$$\lim_{t\to+\infty}\rho(t)\left(\frac{a_1(t)}{a_2(t)}\right)^{-\frac{1+\lambda_1}{2+\lambda_1+\lambda_2}}=\lim_{t\to+\infty}\left(\frac{a_2(t)}{a_1(t)}\frac{|u_1(t)|^{1+\lambda_1}}{1+\lambda_2}+\frac{|u_2(t)|^{1+\lambda_1}}{1+\lambda_1}\right)=+\infty.$$

Nevertheless, this is contradictory to

$$\left(\frac{a_2(t)}{a_1(t)}\frac{|u_1(t)|^{1+\lambda_2}}{1+\lambda_2}+\frac{|u_2(t)|^{1+\lambda_1}}{1+\lambda_1}\right)'=\left(\frac{a_2(t)}{a_1(t)}\right)'\frac{|u_1(t)|^{1+\lambda_2}}{1+\lambda_2}$$

$$=-\left(\frac{a_2(t)}{a_1(t)}\right)^2\left(\frac{a_1(t)}{a_2(t)}\right)'\frac{|u_1(t)|^{1+\lambda_2}}{1+\lambda_2}\le 0.$$

The theorem is proved.

Theorem 2.2. *Suppose that the function* $\dfrac{a_1(t)}{a_2(t)}$ *is nonincreasing and the functions f(t) and g(t)* are *defined by* (2.1) *and* (2.2) *respectively. If the condition*

$$\int_0^{+\infty}|df(t)|<+\infty \quad \text{for} \quad \lambda_1\lambda_2<1,$$

$$\int_0^{+\infty}|dg(t)|<+\infty \quad \text{for} \quad \lambda_1\lambda_2>1,$$

is satisfied then any nontrivial solution of the system (2) *may be represented as* (1.8), *where the: function $\rho(t)$ defined by* (1.7) *and the function $\alpha(t)$ satisfy the. condition* (1.9) *and* $(\omega_1(t)\omega_2(t))$ *is a solution of the problem* (1.10).

Proof. We distinguish two cases, when $\lambda_1\lambda_2$ <l and $\lambda_1\lambda_2$ >1. In the former case, we have

$$\lim_{t\to+\infty}f(t)=\frac{\lambda_1+1}{\lambda_1}\delta_f\le 0.$$

In addition to

$$0\le\lim_{t\to+\infty}\frac{\left(\dfrac{a_1(t)}{a_2(t)}\right)^{\frac{\lambda_1}{1+\lambda_1}}}{\int_0^t a_1(s)ds}=\frac{\lambda_1}{1+\lambda_1}\lim_{t\to+\infty}f(t)=\delta_f.$$

Therefore, $\lim\limits_{t\to+\infty}\beta(t)=0$. As in the proof of the previous theorem, we can prove that conditions (1.1), (1.2) and the estimate (2.5) are satisfied. Supposing that an arbitrary solution ($u_1(t)$, $u_2(t)$) of the system (2) satisfies the condition (1.6) we obtain the inequality

$$\rho(t)\ge 2^{\frac{(1+\lambda_1)(1+\lambda_2)+2+\lambda_1+\lambda_2}{\lambda_1\lambda_2-1}}(1+\lambda_1)^{\frac{1+\lambda_2}{\lambda_1\lambda_2-1}}(1+\lambda_2)^{\frac{1+\lambda_1}{\lambda_1\lambda_2-1}}\left(\frac{3}{2+\lambda_1+\lambda_2}\right)^{\frac{(1+\lambda_1)(1+\lambda_2)}{\lambda_1\lambda_2-1}}$$

$$\left(\frac{a_1(t)}{a_2(t)}\right)^{-\frac{1+\lambda_2}{2+\lambda_1+\lambda_2}}\left(\int_t^{+\infty}|df(\tau)|\right)^{\frac{(1+\lambda_1)(1+\lambda_2)}{\lambda_1\lambda_2-1}},$$

which implies

$$\lim_{t\to+\infty} \rho(t)\left(\frac{a_1(t)}{a_2(t)}\right)^{\frac{1+\lambda_2}{2+\lambda_1+\lambda_2}} = \lim_{t\to+\infty}\left(\frac{|u_1(t)|^{1+\lambda_1}}{1+\lambda_2}+\frac{a_1(t)}{a_2(t)}\frac{|u_2(t)|^{1+\lambda_1}}{1+\lambda_1}\right) = +\infty.$$

We have just got all contradiction, because of initial conditions we have

$$\left(\frac{|u_1(t)|^{1+\lambda_2}}{1+\lambda_2}+\frac{a_1(t)}{a_2(t)}\frac{|u_2(t)|^{1+\lambda_1}}{1+\lambda_1}\right)' = \left(\frac{a_1(t)}{a_2(t)}\right)'\frac{|u_2(t)|^{1+\lambda_1}}{1+\lambda_1} \le 0$$

According to Theorem 1.1. the solution ($u_1(t)$, $u_2(t)$) of the system (2) satisfies the condition (1.4) and the conclusion follows by Theorem 1.2. In the latter case, we have

$$\lim_{t\to+\infty} g(t) = (1+\lambda_2)\delta_g \le 0$$

and

$$0 \le \lim_{t\to+\infty}\frac{\left(\frac{a_1(t)}{a_2(t)}\right)^{\frac{1}{1+\lambda_2}}}{\int_0^t a_1(s)ds} = \frac{1}{1+\lambda_1}\lim_{t\to+\infty} f(t) = \delta_g .$$

Hence $\lim_{t\to+\infty} g(t) = 0$. Consequently, the conditions of Theorem 1.1. are satisfied and the inequality (2.6) is valid. Suppose that all arbitrary solution ($u_1(t)$, $u_2(t)$) of the system (2) satisfies the condition (1.5) which together with (2.6) implies

$$\rho(t) \le 2^{\frac{(1+\lambda_1)(1+\lambda_2)+2+\lambda_1+\lambda_2}{\lambda_1\lambda_2-1}}(1+\lambda_1)^{\frac{1+\lambda_2}{\lambda_1\lambda_2-1}}(1+\lambda_2)^{\frac{1+\lambda_1}{\lambda_1\lambda_2-1}}\left(\frac{3}{2+\lambda_1+\lambda_2}\right)^{\frac{(1+\lambda_1)(1+\lambda_2)}{\lambda_1\lambda_2-1}}$$

$$\left(\frac{a_1(t)}{a_2(t)}\right)^{\frac{1+\lambda_1}{2+\lambda_1+\lambda_2}}\left(\int_t^{+\infty}|dg(\tau)|\right)^{\frac{(1+\lambda_1)(1+\lambda_2)}{\lambda_1\lambda_2-1}},$$

Accordingly,

$$\lim_{t\to+\infty}\rho(t)\left(\frac{a_1(t)}{a_2(t)}\right)^{-\frac{1+\lambda_1}{2+\lambda_1+\lambda_2}} = \lim_{t\to+\infty}\left(\frac{a_2(t)}{a_1(t)}\frac{|u_1(t)|^{1+\lambda_1}}{1+\lambda_2}+\frac{|u_2(t)|^{1+\lambda_1}}{1+\lambda_1}\right) = 0.$$

On the other hand, using the initial conditions, we have

$$\left(\frac{a_2(t)}{a_1(t)}\frac{|u_1(t)|^{1+\lambda_2}}{1+\lambda_2}+\frac{|u_2(t)|^{1+\lambda_1}}{1+\lambda_1}\right)' \ge 0.$$

The obtained contradiction completes the proof by the application of Theorem 1.1. and Theorem 1.2., as in the previous case.

Remark. Taking $\lambda_1 = 1$, $a_1(t)\equiv 1$, $\lambda_2 = \lambda$, $a_2(t) = -p(t)$ from the proved Theorem 2.1. and Theorem 2.2. we obtain respectively Theorem 20.5. and Theorem 20.4. in [4].

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Асимптотическое представление решений системы дифференциальных уравнений типа Эмдена-Фаулера

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Устанавливаются достаточные условия асимптотического представления решений системы дифференциальных уравнений типа Эмдена-Фаулера. Полученные результаты обобщают результаты И.Т.Кигурадзе и Т.А.Чантурия для дифференциального уравнения второго порядка типа Эмдена-Фаулера.

