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РАСЧЕТ ВЫСШИХ ПРОИЗВОДНЫХ ДЛЯ ТРАЕКТОРИЙ РЕЛЯТИВИСТСКИХ ЭЛЕКТРОНОВ

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The theory and experimental researches of radiation of multy GeV electrons in oriented single crystals are now in quick progress (see the topical issue [1]). The so called "synchrotron approximation" (SA) [2] provides much simpler description of the photon emission process than that given by the precise radiation formulas, but the question arises how good such the approximation is. The corrections to the SA can be expressed in terms of the high derivatives of the electron trajectory [3]. For example, the phase factor in the radiation formulas (see exp. (1.37) in ref. [4] and also exps. (3), (10) in ref. [5]) may be presented in powers of time τ as

$$\Delta_s = \omega\,(\tau - \frac{1}{c}\left|\vec{r}_+ - \vec{r}_-\right|\,) \tag{1}$$

$$\approx \omega\left[\frac{\tau}{2\gamma^2} + \frac{\dot{\vec{\beta}}^2\tau^3}{24} + \left(\frac{\ddot{\vec{\beta}}^2}{1440} + \frac{\dot{\vec{\beta}}\dddot{\vec{\beta}}}{480}\right)\tau^5\right], \tag{2}$$

where $\vec{v} = c\vec{\beta}$ is the electron velocity, $\vec{r}_+ = \vec{r}(t_0 + \tau/2)$, $\vec{r}_- = \vec{r}(t_0 - \tau/2)$, $\vec{r}(t)$ is the electron trajectory, $\dot{\vec{\beta}} = d\vec{\beta}/dt$ is the acceleration associated with the instantaneous angular frequency of the electron $\Omega = \left|\dot{\vec{\beta}}\right|$, $\ddot{\vec{\beta}} = d^2\vec{\beta}/dt^2$ and $\dddot{\vec{\beta}} = d^3\vec{\beta}/dt^3$, γ is the Lorentz factor. The SA corresponds to the first two terms in the square brackets of eq.(2).

The purpose of the present paper is to express the quantities containing the high time derivatives like $\dot{\vec{\beta}}\dddot{\vec{\beta}}$ in eq.(2) through the integrals of motion and the distance to the external field centre (i.e. atomic axes and planes).

The trajectory of ultra-relativistic electron can be devided into the transverse and longitudinal parts (the latter is assumed to coincide with the atomic axis, i.e. with z-direction) [5]. Neglectind the terms $\sim \vec{\beta}_\perp^4$ one can get

$$\begin{aligned}
\vec{\beta} &= (\vec{\beta}_\perp, 1 - \gamma^{-2}/2 - \vec{\beta}_\perp^2/2, \\
\dot{\vec{\beta}} &= (\dot{\vec{\beta}}_\perp, -\vec{\beta}_\perp\dot{\vec{\beta}}_\perp, \\
\ddot{\vec{\beta}} &= (\ddot{\vec{\beta}}_\perp, -\dot{\vec{\beta}}_\perp^2 - \vec{\beta}_\perp\ddot{\vec{\beta}}_\perp), \\
\dddot{\vec{\beta}} &= (\dddot{\vec{\beta}}_\perp, -3\dot{\vec{\beta}}_\perp\ddot{\vec{\beta}}_\perp - \vec{\beta}_\perp\dddot{\vec{\beta}}_\perp), \\
\ddddot{\vec{\beta}} &= (\ddddot{\vec{\beta}}_\perp, -3\ddot{\vec{\beta}}_\perp^2 - 4\dot{\vec{\beta}}_\perp\dddot{\vec{\beta}}_\perp - \vec{\beta}_\perp\ddddot{\vec{\beta}}_\perp),
\end{aligned} \tag{3}$$

The first two relations give

$$\vec{\beta}\dot{\vec{\beta}} = \frac{1}{2}(\vec{\beta}_\perp \dot{\vec{\beta}}_\perp)(\gamma^{-2} + \vec{\beta}_\perp^2). \tag{4}$$

It follows from eq.(4) that the acceleration of ultrarelativistic electron $(\gamma >> 1)$ is perpendicular to the velocity with the accuracy of $\sim \beta_\perp^3$. We shall therefore assume that $\beta\dot{\beta} = 0$. The latter equality gives the relations

$$\dot{\vec{\beta}}^2 + \vec{\beta}\ddot{\vec{\beta}} = 0$$

$$3\dot{\vec{\beta}}\ddot{\vec{\beta}} + \vec{\beta}\dddot{\vec{\beta}} = 0, \tag{5}$$

$$3\ddot{\vec{\beta}}^2 + 4\dot{\vec{\beta}}\dddot{\vec{\beta}} + \vec{\beta}\vec{\beta}^{(4)} = 0$$

where all the time derivatives of the velocity can be replaced by their projections on the transverse plane with the accuracy of $\sim \beta_\perp$. This is because of the simple estimation for n'th time derivative [3] $\beta_z^{(n)} / \beta_\perp^{(n)} \sim \beta_\perp$.

For an electron moving in a continuum potential of atomic axis (planes) $U(\vec{\rho})$ the longitudinal momentum is constant, and the transverse motion satisfies the equations of motion

$$mc\gamma\dot{\vec{\beta}}_\perp = -\nabla U, \quad mc\gamma\beta_z = const, \tag{6}$$

where $\nabla = \partial / \partial\vec{\rho}$, $\vec{\rho}$ is the transverse coordinate such that c $\vec{\beta} = \dot{\vec{\rho}}$.

Consider now the important case of axially symmetrical axis potential $U(\vec{\rho}) = U(\rho)$. Introducing the transverse polar coordinates ρ and φ we get

$$\vec{\beta} = c^{-1}\dot{\vec{\rho}} = c^{-1}(\dot{\rho}, \rho\dot{\varphi}), \quad \dot{\vec{\beta}} = c^{-1}\ddot{\vec{\rho}} = c^{-1}(\ddot{\rho} - \rho\dot{\varphi}^2, \rho\ddot{\varphi} + 2\dot{\rho}\dot{\varphi}), \tag{7}$$

The conservation of the angular momentum μ with respect to the string yields

$$\mu = m\gamma\rho^2\dot{\varphi} = const, \quad \rho\ddot{\varphi} + 2\dot{\rho}\dot{\varphi} = 0, \tag{8}$$

The first equation in (6) has now the form

$$m\gamma\ddot{\rho} = -U' + \mu^2 / (m\gamma\rho^3) \equiv -U_e' \tag{9}$$

where $U' \equiv \dfrac{dU}{d\rho}, U_e = U + \dfrac{\mu^2}{(2m\gamma\rho^2)}$ - is the effective potential.

One can also obtain

$$\dot{\varphi} = \mu / (m\gamma\rho^2), \quad \dot{\rho} = \pm\sqrt{2(\varepsilon - U_e) / (m\gamma)}, \tag{10}$$

where ε is the transverse energy.

One can now express all the time derivatives of the velocity in terms of the transverse energy ε and the angular momentum μ for given distance to the string ρ. We shall use the relations

$$\ddot{\varphi} = -\frac{2\mu\dot{\rho}}{m\gamma\rho^3}, \quad \dddot{\varphi} = \frac{6\mu\dot{\rho}^2}{m\gamma\rho^4} - \frac{2\mu\ddot{\rho}}{m\gamma\rho^3} \tag{11}$$

$$\dddot{\rho} = -U_e''\frac{\dot{\rho}}{m\gamma}, \dots,$$

Taking into account eqs.(11) we finally get

$$\dot{\vec{\beta}}\ddot{\vec{\beta}} = \frac{c^2}{E^2}\dot{\rho}U'U''$$

$$\dot{\vec{\beta}}\dddot{\vec{\beta}} = -\frac{c^2}{E^2}U'\left[\frac{c^2}{E}\left(U'U'' + \frac{\mu^2c^2}{E\rho^4}U' - \frac{\mu^2c^2}{E\rho^3}U''\right) - \dot{\rho}^2U'''\right], \qquad (12)$$

$$\ddot{\vec{\beta}}^2 = -\frac{c^4}{E^3}\left[2(\varepsilon - U)(U'')^2 - \frac{\mu^2c^2}{E\rho^2}(U'')^2 + \frac{\mu^2c^2}{E\rho^4}(U')^2\right],$$

To obtain the expressions for planar channeling one has to assume $\mu = 0$ in eqs.(12).

Formulas (12) represent the final result of the paper, namely, for given distance to the atomic string the high derivative products in eq.(2) are expressed in terms of the integrals of transverse motion ε and μ.

References

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CALCULATION OF HIGH DERIVATIVES FOR RELATIVISTIC ELECTRON TRAJECTORIES

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