ON GLOBAL CONSTANT SIGN SOLUTIONS OF ONE SYSTEM DIFFERENTIAL INEQUALITIES

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Let us consider systems of inequalities

$$u_2' sign u_1 \leqslant -\alpha_2(t)|u_1|^{\lambda_2} \leqslant 0 \leqslant \alpha_1(t)|u_2|^{\lambda_1} \leqslant u_1' sign u_2 \tag{1}$$

and

$$-\alpha_2(t)|u_1|^{\lambda_2} \leqslant u_2' sign u_1 \leqslant 0 \leqslant u_1' sign u_2 \leqslant \alpha_1(t)|u_2|^{\lambda_1} \tag{2}$$

where $\alpha_i : (0,+\infty) \to [0,+\infty) (i=1,2)$ are summable on each segment from $(0,+\infty)$ function, $\lambda_i > 0 (i=1,2)$.

The present contribution gives the precise requirements guaranteeing lack of global constant sign solutions at systems of an aspect (1), that is such solutions that $u_1(t) \cdot u_2(t) \neq 0$ at $0 < t < +\infty$. The similar conditions for a case, when the argument t is in a neighborhood $+\infty$, are available in the monograph [1].

Theorem 1. *Let $\lambda_1 \cdot \lambda_2 < 1$ and for some $t_0 \in (0,+\infty)$*

$$\int_0^{t_0} \alpha_1(t)dt < +\infty, \int_0^{t_0} \alpha_2(t)dt = +\infty, \int_{t_0}^{+\infty} \alpha_1(t)dt = +\infty, \int_{t_0}^{+\infty} \alpha_2(t)dt < +\infty. \tag{3}$$

Then if

$$\int_0^{t_0} \alpha_1(t)\Big(\int_t^{t_0} \alpha_2(\tau)d\tau\Big)^{\lambda_1} dt + \int_{t_0}^{+\infty} \alpha_2(t)\Big(\int_{t_0}^{t} \alpha_1(\tau)d\tau\Big)^{\lambda_2} dt = +\infty, \tag{4}$$

the system (1) *has not global solutions, such that $u_1(t) \cdot u_2(t) > 0$ at $0 < t < +\infty$.*

Theorem 2. *Let $\lambda_1 \cdot \lambda_2 < 1$ and* (3) *has a place. If the system* (2) *has not global solutions, such that $u_1(t) \cdot u_2(t) > 0$ at $0 < t < +\infty$, then the equality* (4) *is carried out.*

Theorem 3. *Let $\lambda_1 \cdot \lambda_2 < 1$ and for some $t_0 \in (0,+\infty)$*

$$\int_0^{t_0} \alpha_1(t)dt = +\infty, \int_0^{t_0} \alpha_2(t)dt < +\infty, \int_{t_0}^{+\infty} \alpha_1(t)dt < +\infty, \int_{t_0}^{+\infty} \alpha_2(t)dt = +\infty, \tag{5}$$

Then if

$$\int_0^{t_0} \alpha_2(t)\Big(\int_t^{t_0} \alpha_1(\tau)d\tau\Big)^{\lambda_2} dt + \int_{t_0}^{+\infty} \alpha_1(t)\Big(\int_{t_0}^{t} \alpha_2(\tau)d\tau\Big)^{\lambda_1} dt = +\infty, \tag{6}$$

the system (1) has not global solutions, such that $u_1(t) \cdot u_2(t) < 0$ at $0 < t < +\infty$.

Theorem 4. *Let $\lambda_1 \cdot \lambda_2 < 1$ and (5) has a place. If the system (2) has not global solutions, such that $u_1(t) \cdot u_2(t) < 0$ at $0 < t < +\infty$, then the equality (6) is carried out.*

Theorem 5. *Let $\lambda_1 \cdot \lambda_2 = 1$, (3) has a place and the requirement*

$$\lim_{n\to+\infty} \Big(\int_0^{t_0} \alpha_2(\tau)\rho_n^{1+\lambda_2}(\tau)d\tau + \int_{t_0}^{+\infty} \alpha_1(\tau)r_n^{1+\lambda_1}(\tau)d\tau\Big) < +\infty \tag{7}$$

is respected, where

$$r_0(t)=\int\limits_t^{+\infty}\alpha_2(\tau)d\tau, r_n(t)=\lambda_2\int\limits_t^{+\infty}\alpha_1(\tau)r_{n-1}^{1+\lambda_1}(\tau)d\tau+\int\limits_t^{+\infty}\alpha_2(\tau)d\tau, t_0\leqslant t<+\infty,$$

$$\rho_0(t)=\int\limits_0^{t}\alpha_1(\tau)d\tau, \rho_n(t)=\lambda_1\int\limits_0^{t}\alpha_2(\tau)\rho_{n-1}^{1+\lambda_2}(\tau)d\tau+\int\limits_0^{t}\alpha_1(\tau)d\tau, 0<t\leqslant t_0.$$

Then if

$$r(t_0)\cdot\rho^{\lambda_2}(t_0)>1, \tag{8}$$

where $r(t)=\lim\limits_{n\to+\infty}r_n(t), \rho(t)=\lim\limits_{n\to+\infty}\rho_n(t)$, *the system (1) has not global solutions, such that* $u_1(t)\cdot u_2(t)>0$ *at* $0<t<+\infty$.

Theorem 6. *Let* $\lambda_1\cdot\lambda_2=1$, *and (3) and (7) take place. If the system (2) has not global solutions, such that* $u_1(t)\cdot u_2(t)>0$ *at* $0<t<+\infty$, *then the inequlity (8) is valid.*

Theorem 7. *Let* $\lambda_1\cdot\lambda_2=1$, *(5) has a place and the requirement*

$$\lim_{n\to+\infty}\left(\int\limits_0^{t_0}\alpha_1(\tau)\rho_n^{1+\lambda_1}(\tau)d\tau+\int\limits_{t_0}^{+\infty}\alpha_2(\tau)r_n^{1+\lambda_2}(\tau)d\tau\right)<+\infty \tag{9}$$

is respected, where

$$r_0(t)=\int\limits_t^{+\infty}\alpha_1(\tau)d\tau, r_n(t)=\lambda_1\int\limits_t^{+\infty}\alpha_2(\tau)r_{n-1}^{1+\lambda_2}(\tau)d\tau+\int\limits_t^{+\infty}\alpha_1(\tau)d\tau, t_0\leqslant t<+\infty,$$

$$\rho_0(t)=\int\limits_0^{t}\alpha_2(\tau)d\tau, \rho_n(t)=\lambda_2\int\limits_0^{t}\alpha_1(\tau)\rho_{n-1}^{1+\lambda_1}(\tau)d\tau+\int\limits_0^{t}\alpha_2(\tau)d\tau, 0<t\leqslant t_0.$$

Then if

$$r(t_0)\cdot\rho^{\lambda_1}(t_0)>1, \tag{10}$$

where $r(t)=\lim\limits_{n\to+\infty}r_n(t), \rho(t)=\lim\limits_{n\to+\infty}\rho_n(t)$, *the system (1) has not global solutions, such that* $u_1(t)\cdot u_2(t)<0$ *at* $0<t<+\infty$.

Theorem 8. *Let* $\lambda_1\cdot\lambda_2=1$, *and (5) and (9) have a place. If the system (2) has not global solutions, such that* $u_1(t)\cdot u_2(t)<0$ *at* $0<t<+\infty$, *then the inequality (10) is valid.*

Reference

1. *Mirsov J.D.* Asymptotic properties of solutions of systems of the nonlinear nonautonomous ordinary differential equations. Maikop, RIPO "Adygeia", 1993, pp.132 (in Russian).

О глобальных знакопостоянных решениях одной системы дифференциальных неравенств

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Приведены точные условия, гарантирующие отсутствие глобальных знакопостоянных решений одной системы дифференциальных неравенств.

