ON PRINCIPAL SOLUTIONS OF ONE SYSTEM OF THE NONLINEAR DIFFERENTIAL EQUATIONS

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Resume

Consinder a system

$$u_1' = a_1(t)|u_2|^{\lambda_1} sign u_2, u_2' = -a_2(t)|u_1|^{\lambda_2} sign u_1, \tag{1}$$

where $a_1 : [0, +\infty) \to [0, +\infty), a_2 : [0, +\infty) \to R$ are locally summable functions, $\lambda_i > 0 (i = 1, 2)$ and

$$\lambda_1 \lambda_2 = 1. \tag{2}$$

Let the solution $u_1(t), u_2(t)$ of system (1) is those, that for some $t_0 \geq 0, u_1(t) \neq 0$ at $t \geq t_0$. Let's assume

$$z(t) = u_2(t) sign u_1(t)/|u_1(t)|^{\lambda_2}\, for\, t \geq t_0. \tag{3}$$

It is easily tested that z(t) is a solution of the following equation of the Riccati type

$$z' + \lambda_2 a_1(t)|z|^{1+\lambda_1} + a_2(t) = 0 \tag{4}$$

given on $[t_0, +\infty)$.

Let there is $t_1 > t_0$ such that $\int\limits_{t_0}^{t_1} a_1(t)dt > 0$. Then we'll find $t^0 > t_0$ and that equation (4) has a solution, given on $[t_0, t^0)$ and not on to the right.

We shall designate as M a point set of straight $t = t_0$ though which solutions not gone on to the right pass. Clearly, that this set is bounded above. Let's assume $\gamma = sup M$ and following [1] we shall introduce definition.

Definition 1. *We shall call solution $z(t)$ defined at initial condition $z(t_0) = \gamma$ a* **boundary** *solution of equation* (4).

Definition 2 (see [2]). *We shall call solution $u_1(t), u_2(t)$ of system* (1) *with a component $u_1(t) \neq 0$ for $t \geq t_0$ the* **principal**, *if the function $z(t)$, defined by equality* (3), *is a boundary solution of equation* (4).

Theorem. *Let* (2),

$$\int\limits_{t_0}^{+\infty} a_1(t)dt = +\infty, 0 \leq \int\limits_{t}^{+\infty} a_2(\tau)d\tau < +\infty$$

$$r_0(t) = \int\limits_{t}^{+\infty} a_2(\tau)d\tau, r_n(t) = \lambda_2 \int\limits_{t}^{+\infty} a_1(\tau)r_{n-1}^{1+\lambda_1}(\tau)d\tau + \int\limits_{t}^{+\infty} a_2(\tau)d\tau,$$

$$\lim_{n \to +\infty} r_n(t) = r(t)$$

at $t \geq t_0$ take place. Then in order that the solution $u_1(t), u_2(t)$ of system (1) *may be the principal, realization of equality*

$$u_2(t) sign u_1(t)/|u_1(t)|^{\lambda_2} = r(t)\ for\ t \geq t_0$$

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is necessary and sufficiently or, that same,

$$u_1(t) = u_1(t_0)\exp\int\limits_{t_0}^{t} a_1(\tau)r^{\lambda_1}(\tau)d\tau \ for \ t \geq t_0$$

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References

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О главных решениях одной системы нелинейных дифференциальных уравнений

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Указана оригинальная характеристика главного решения одной нелинейной системы дифференциальных уравнений.

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